Assessing Students' Meaningful Learning through a Paper and Pen Test

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Students' meaningful learning can be assessed in a paper and pen test by deliberately asking questions that require them to indicate deep understanding of complex ideas that are relevant or real life. These questions are of higher order thinking skills outlined in the revised Bloom's Taxonomy of Educational Objectives. This study aimed to describe the indicators for students' meaningful learning after they had gone through a series of group activities where they discuss and engage in meaning making of an authentic task involving modeling of linear equations and functions. A six-item paper and pen test with open ended questions and problems was administered to a class of freshmen taking Bachelor of Science in Education major in Mathematics. In depth analysis of four selected students' written solutions and explanations was used to describe their meaningful learning. Results indicated that three of the four students had positive learning outcomes than the errors they committed which were mostly due to careless miscalculations. Apart from doing menial tasks as graphing and doing computations, students were challenged to give reasons on the mathematical processes they needed to go through and apply what they learned in different contexts.

Keywords: Meaningful learning, deep understanding, linear equations, functions

Meaningful learning is achieving deep understanding of complex ideas that are relevant to students' lives (Jonassen, Peck and Wilson, 1999). Kierkegaard posits that "In order to help our students to learn effectively, we teachers must understand what he understands… and in the way he understands it" (cited in Kegan, 1994, p.278). This means, more than being able to see if a student gets the correct answer or not, teachers should know students' thinking and reasoning skills on a concept so that misconceptions could be corrected or student's correct novel approaches could also be encouraged.

The revised Bloom's Taxonomy of Educational Objectives (Anderson & Krathwohl, 2001) can be used to assess the meaningful learning of students (Clark, 2002). Meaningful learning occurs if students are able to apply higher order thinking skills in solving problems, in explaining or elaborating on Algebra concepts. These higher order thinking skills (as outlined in the revised Bloom's Taxonomy of Educational Objectives) are apply, analyze, evaluate, and create.

This study is both descriptive and qualitative. It describes what concepts in linear equations and functions students have learned after a series of activities they were engaged in social interaction with one another. The focus of the study involves only four cases, but in depth results of their written work are reported here. This study suggests a way to make meaning how students understand the concepts learned in the class activities. It aims to answer the question "to what extent has meaningful learning occurred after each activity?"

This is a case study of four students who were selected on the basis of their self-efficacy (confidence) levels in Algebra. The four cases selected included two students identified as having the lowest self-efficacy and two students having the highest self-efficacy. Students' profile as to their background in their high school mathematics was gathered at the start of the term: name, type of high school where they graduated from, and their age. Careful scrutiny and analysis for the reporting of the results in the study was anchored on these four cases.

The research question posed was systematically pursued using the basic interpretative qualitative research design. Qualitative data for the students' perceptions, insights, and knowledge claims as they engaged in collaborative metacognitive activities were collected from the solutions and answers in paper and pen tests which were carefully analyzed for their meaningful learning. The detailed description of meaningful learning necessitated the use of specific cases. Hence, a multiple case study research design was also employed.

Meaningful Learning

To measure students' meaningful learning, mean scores from the three pencil and paper tests were used. The researcher relied on the solutions and answers students indicated to illustrate whether they can apply, analyze, evaluate and create as defined in the revised Bloom's Taxonomy of Educational Objectives. Success in these test items indicated that students have demonstrated higher levels of cognitive functioning in contrast to rote learning.

Meaningful Learning from Students' Paper and Pen Tests

Table 1
Students' Paper and Pen Test Scores in Percentages

	Linear Equations and Functions
Patricia	96
Ria	80
Michael	90
Sara	62

Although, in as much as the teacher who rated these papers tried his best to be objective, judging from these averages, both Michael and Sara demonstrated satisfactory understanding of the necessary concepts in linear equations and functions. Looking at Ria's and Sara's quizzes, one may doubt if they really had meaningful learning because these scores were satisfactory. Clearly, Ria and Sara's low scores in their tests show that they were not able to exhibit higher levels of cognition. Therefore, these test scores were not sufficient to show that Ria and Sara had meaningful learning. To a limited extent, it can be deduced that they had somewhat meaningful learning.

Paper and pen test on linear equations and functions (first metacognitive activity).

Patricia was able to relate the words “uphill” and “downhill” to slope of lines. She creatively used directions (southwest to northeast and northwest to southeast) and drawings of a hiker to illustrate her point. Clearly she understood the concept of slope. She was able to explain vividly in her own way how the slope can be related to a different context such as climbing up and down a hill. Her answer was considered the best in this item in the class. Please see the next figure.

Patricia correctly answered all items except a mistake in the graph. Patricia sketched the graph correctly with the use of the x and y intercepts, and labeled them properly but failed to check if all values in the line are meaningful. She included even the negative values in the graph when the quantities in the problem represent only positive values.

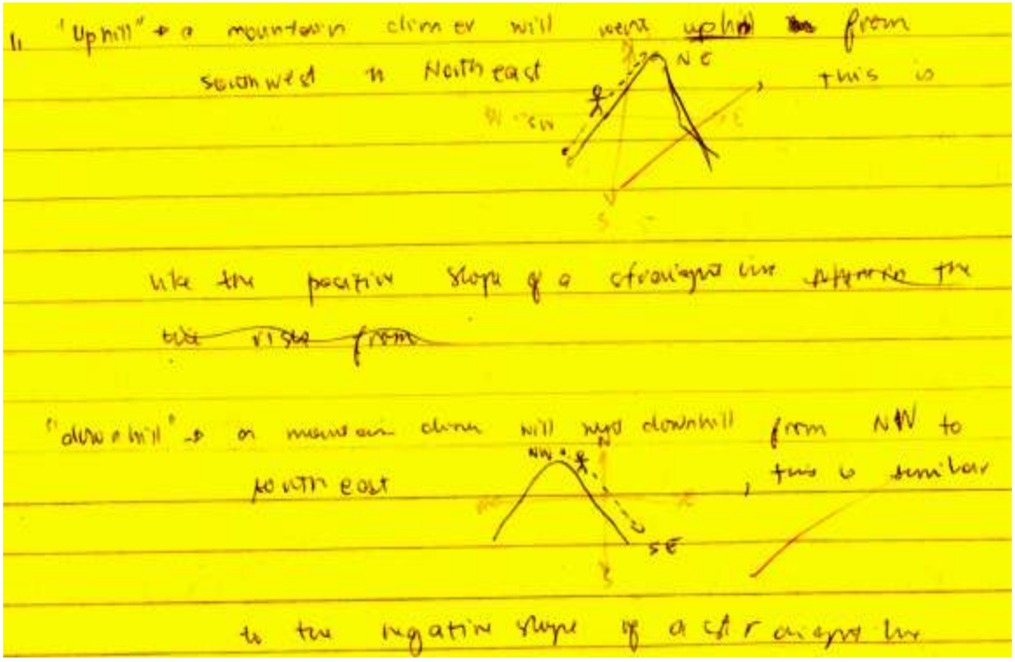

Figure 1. Patricia’s answer in quiz 1 item 1

Patricia was careful in her calculations in solving the problems. She wrote down given values and labeled them, gave representations of the variables she used in the equation, represented costs, income and profit with correct equations and correctly evaluated for the required quantities asked in the problem. She also understood that rates of change are slopes of a linear function.

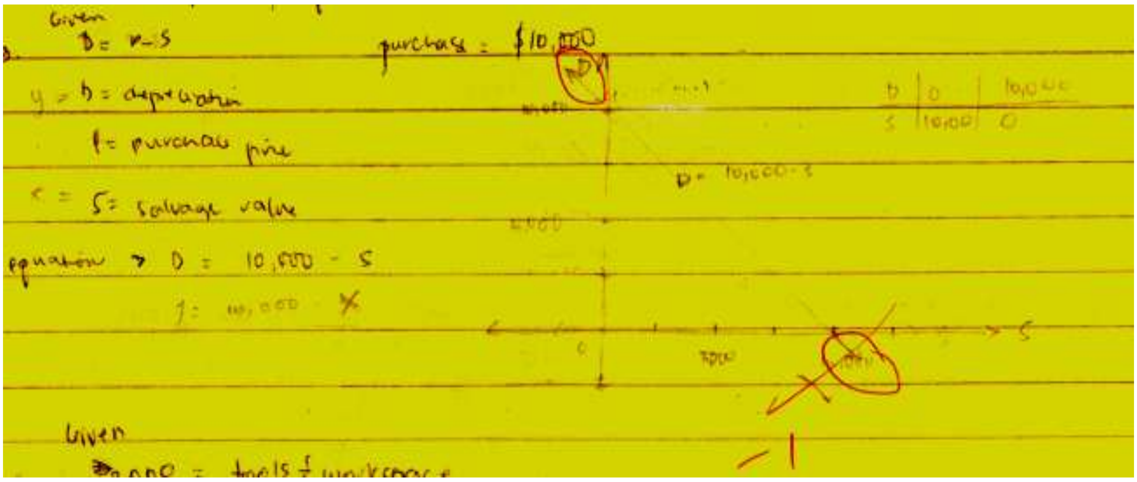

Figure 2. Patricia's answer in quiz 1 item 2

Both Patricia and Ria were able to explain clearly why the *y*-intercept is indicated as the value of *b* in the equation. However, they did not mention the definition of the *y*-intercept instead they made use of the procedure of obtaining the *y*-intercept by setting the *x* value in the equation to zero. Nevertheless, both were able to articulate their reasoning.

Ria gave an example to illustrate her understanding of the slope as it relates to the words "uphill" and "downhill". She made use of analogy of going up and down the hill as going up and down in a Cartesian plane.

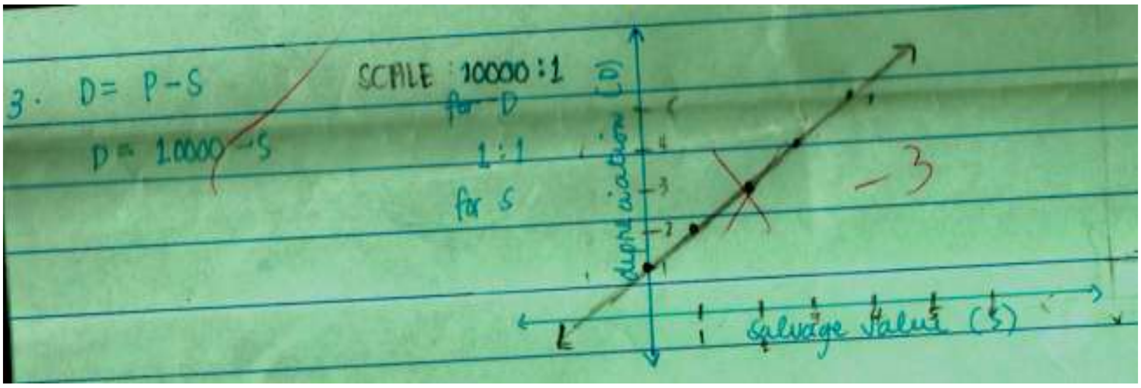

Figure 3. Ria's answer in quiz 1 item 3

Just as Patricia, Ria also included the negative values in the graph which were meaningless values in the problem. On top of that she indicated a scale of 8:1 that only applies to the y-axis. She did not properly label the scale as applying it only on the y-axis. Her graph can be interpreted as applying the scale to values of both the x and y axes. In another item, Ria did not only include the negative values in the graph, she also had a graph that has positive slope when the equation clearly showed a negative slope. The Points in her graph gave values that were inconsistent with their true values in representing equations.

Ria had difficulty in problems involving cost and profit. She did not include the cost of materials needed to make an item unit in the cost function. Although she correctly represented the income function, her profit function was incorrect because it required the use of the cost function. Consequently, she gave wrong answers in subsequent sub items that made use of the profit function. In her interview, she explained that her lack of exposure and familiarity to this kind of problem had caused her points for her quiz. Please refer to the next figure.

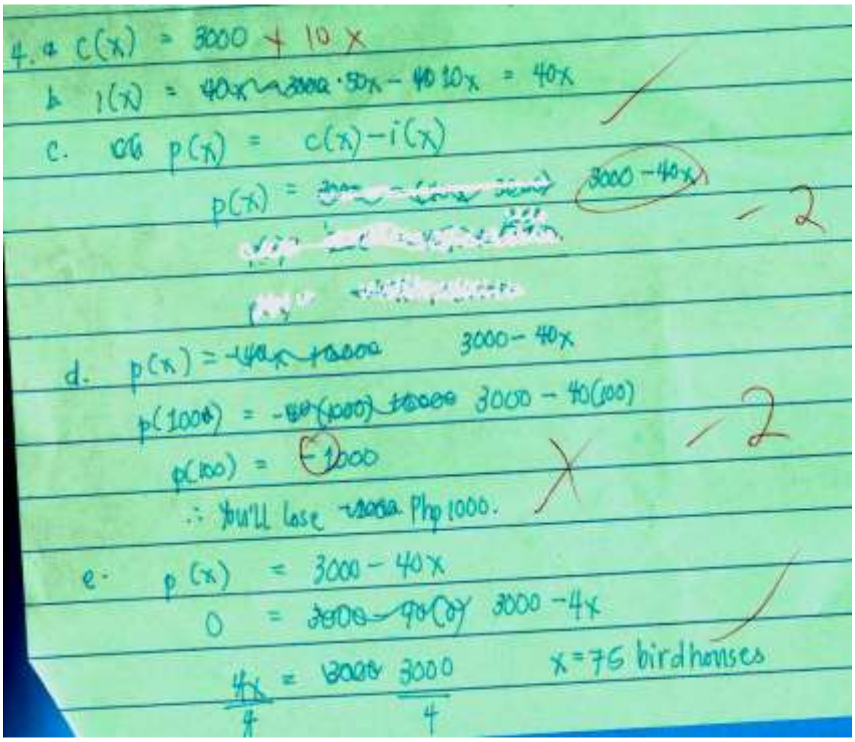

Figure 4. Ria's solution and answer in quiz 1 item 4

In another item, Ria listed the given values in a tabular form. She also made use of the two-point formula in writing the linear equation. However, along the way, she made a careless computation. Please refer to the figure shown below.

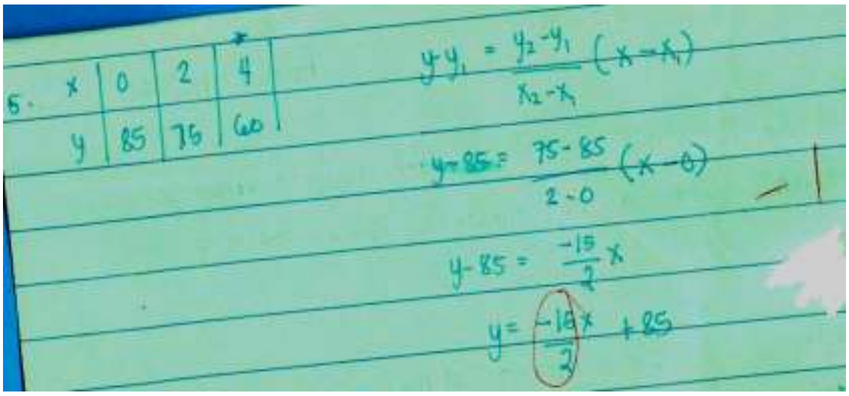

Figure 5. Ria's solution and answer in quiz 1 item 5

Michael related the terms "uphill" and "downhill" to the slope by illustrating directions (going up and right; going down and left) but he incorrectly mentioned downhill as going down and left (refer to Figure 25). Not only was he able to sketch the graph correctly with complete labels, he was also able to consider only realistic values in the graph. He correctly identified what properties of lines (slope, y-intercept, x-intercept, a solution) the given numbers in the problem represent. He correctly gave the functions and used these in finding the required unknown values in the problem. He showed complete solutions and gave answers that were properly labeled.

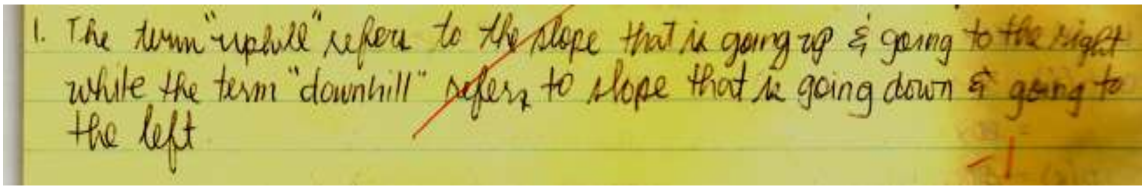

Figure 6. Michael's answer in quiz 1 item 1

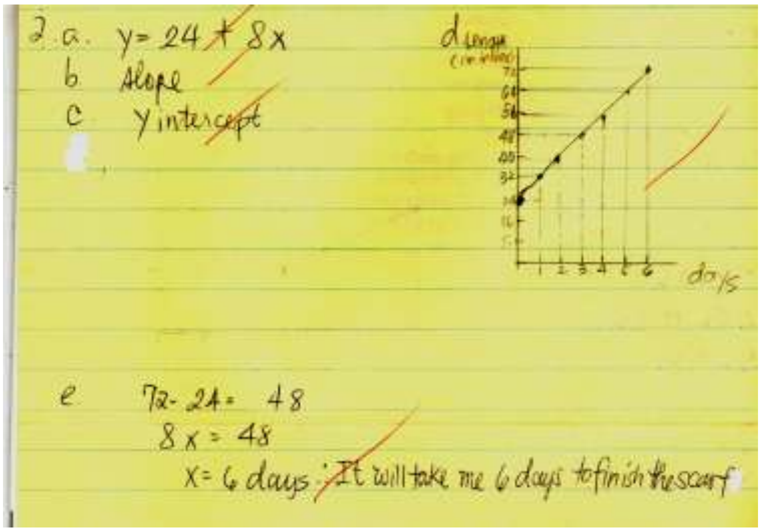

Figure 7. Michael's answer in quiz 1 item 2

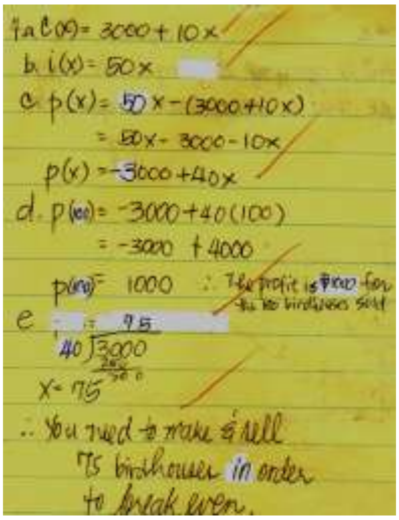

Figure 8. Michael's solution and answer in quiz 1 item 4

When asked to explain why "The *b* in the equation $y = mx + b$ is the *y*-intercept", Michael just repeated what was said in the problem, he wrote "because the slope of the equation is always beside the value of *x*. The *y*-value would always depend on the slope and the *y*-intercept" He knew how to get the slope and the *y*-intercept from a linear equation but he cannot explain why such were the representations in the equation. He was not able to make a link between the concept and his procedural knowledge.

Sara was able to relate hills to linear functions in that she wrote “hills have slopes just like linear functions. The line is formed depending on its slope. When you are climbing a mountain going up so the same as” But she was not able to differentiate an uphill from a downhill which was demanded in the question as implied by the phrase “that these depend on where you are standing on a hill and which way you are looking”

As with Ria and Patricia, Sara’s graph not only included negative values, the axes were not properly labeled leaving readers to guess what values were represented by the points. The line was also incorrect because it intersected the y-axis at a negative value when the given y-intercept should have been 24 inches.

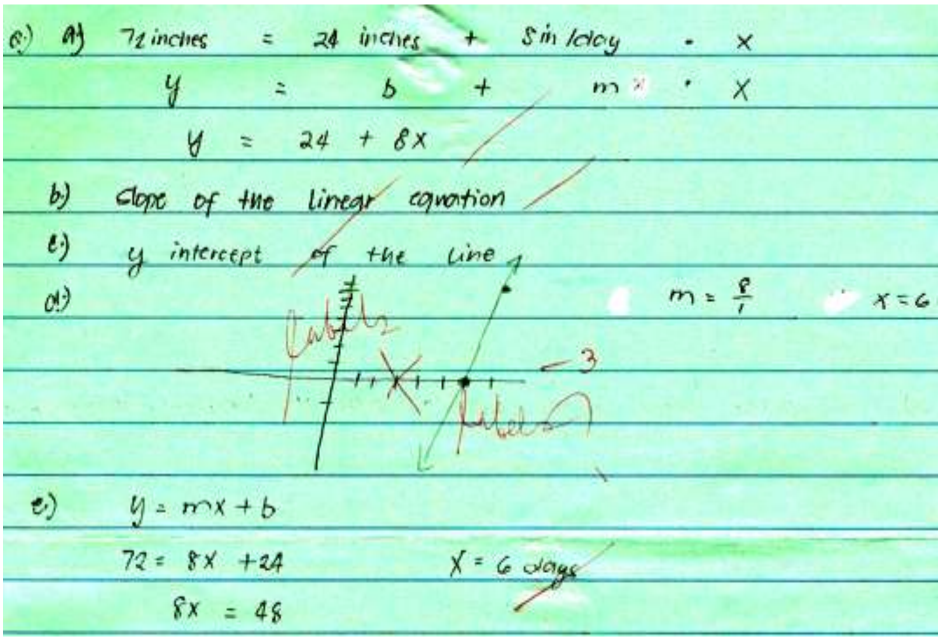

Figure 9. Sara’s answer in quiz 1 item 2

Just like Ria, Sara’s other graph was incorrect in that despite giving the correct equation, she gave a line segment that has positive slope which contradicted the indicated negative slope in the equation. Her solution shows that she got a correct y-intercept but somehow failed to translate this idea into the graph. The graph indicated a zero y-intercept.

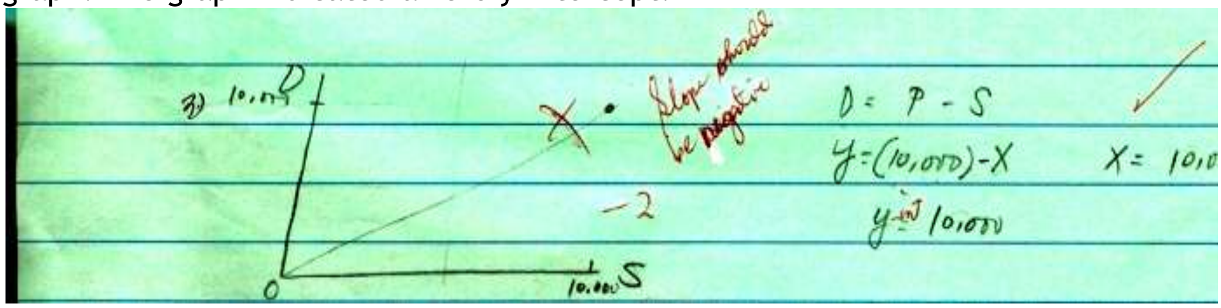

Figure 10. Sara’s answer in quiz 1 item 3

Sara correctly gave the cost and income functions. However, she interchanged the minuend and subtrahend in her profit function. That is, she subtracted the income from the cost instead of the other way around. Thus, her subsequent answers were also incorrect as these made use of the profit function.

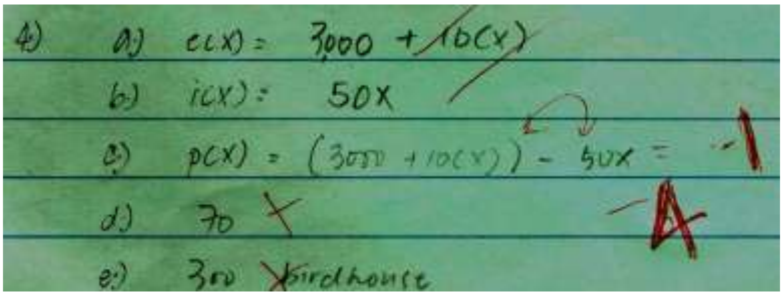

Figure 11. Sara's answers in quiz 1 item 4

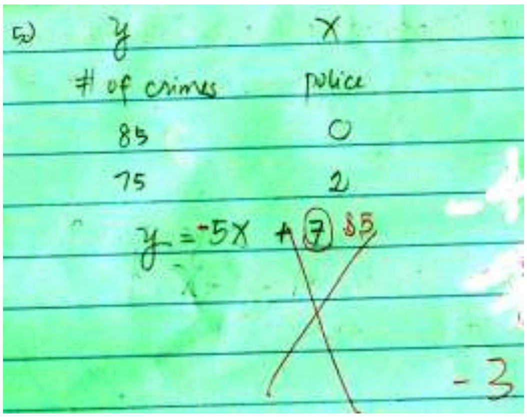

Figure 12. Sara's answer in quiz 1 item 5.

At another item, Sara wrote the variables and their representations. She wrote the given values in tabular form but failed to show any solution on how she was able to write the equation. Her equation indicated an incorrect slope 5 (supposed to be -5) and an incorrect y-intercept 7 (instead of 85). This means she did not know how to use the concept of slope and y-intercept, nor a formula in writing a linear equation as there were no solutions to support this.

When asked to explain why the value *b* in *y* = *mx* + *b* is the y-intercept, Sara wrote "It always depends on the *x* and how the slope moves." She seems to be confused on what the question was asking her to do. Her answer was way too far from the correct answer. Clearly she did not understand what the y-intercept is and why it is designated as b in the equation *y* = *mx* + *b*.

Patricia had deep understanding of the concepts learned in the lesson. She correctly explained what the variables represent and why such variables behave as they do in the equation. She got all items correctly answered except that she failed to consider realistic values in her graphs. This is indicative that meaningful learning had occurred.

Ria's low score can be attributed to her misuse of a scale in her graphs. She was so confident to use a scale factor that unknowingly she incorrectly graphed the linear functions. Furthermore, she gave an incorrect cost function that had affected her subsequent answers. She justified such errors to her lack of exposure to problems of this kind. Nonetheless, she should have taken time to further analyze what the given Php10 variable cost could have meant. She also had a careless computation. If she only took time to evaluate her answers, she might have noticed her errors and had a higher score instead. During the interview, Ria claimed to have learned the lesson from the metacognitive activity. This could be a case where over self confidence can also adversely affect one's performance in that she did not take time to go over her answers once again.

Michael was able to answer all items except the last one where he was asked to explain why in the equation $y = mx + b$, b is the y-intercept of the line. Nonetheless, his correct answers in other items indicated that he was able to apply all the lessons learned in a problem solving situation. This is indicative that meaningful learning had occurred.

Sara's paper and pen test indicated that she did not understand how slopes could indicate the trend of the linear function. She had an incomplete answer in the first question that asked for an explanation on how slopes of linear functions can be compared to a hill. She explained during the interview that she lacked the time in answering the test. On the other hand, Sarah correctly translated two out of three real life problems to a mathematical model using symbols and operations. However, she did not indicate the labels in her graph. There were also computational errors in her solutions. There were items where she was confused which quantities should be substituted to which variables. When asked why in the equation $y = mx + b$, b is the y-intercept, she gave a vague answer, "it depends on x and how the slope moves." Sara had yet to learn a lot of things in this topic. Although, she claimed that the metacognitive activities she did with the group had improved her self-efficacy in linear equations and functions, it looks like because she had low to moderate metacognitive behaviors in this activity that she just had this much understanding of linear equations and functions.

Table 2 summarizes the meaningful learning of the four cases on linear equations and functions.

Meaningful learning was ascertained from their paper and pen tests. Students' paper and pen tests indicated meaningful learning. They had positive learning outcomes than the errors they committed. Their errors were mostly due to careless miscalculations. Students sometimes performed well in their test and at times barely passed. Their performances in their tests made one doubt whether meaningful learning had occurred. Factors such as the inadequate time allotment in the test administration and the difficulty of test were some of the possible reasons. Clearly, meaningful learning required that they expend more efforts in engaging activities for meaning making as much as did Patricia and Michael. They have to take more active part and share of the group work. Since meaningful learning requires exhibiting higher order thinking skills, they should not be contented with the menial tasks as graphing, bringing reference materials, and doing computations. They should challenge themselves to give reasons on the mathematical processes they need to go through, ask more inquiry questions and be involved in seeking for answers to these questions. Going beyond the acquisition of facts require that students can apply what they learned from these lessons in different contexts.

Table 2
Patricia's' Meaningful Learning

Cases	Meaningful Learning
Patricia	(+) Correctly described, applied and used slopes and intercepts in solving problems; (+) Formulated correct linear equations and functions as models of a real-life problem; (+) Correctly identified what the given and unknown quantities represent in the linear equations and graphs; (+) Graphs were sketched correctly except that it inappropriately included negative values for the given problem; (+) Carried out all computations correctly; (+) Explained clearly the reason why the y-intercept can be found as the constant term in the slope-intercept form of the equation;
Ria	(+) Correctly described, applied concepts of slopes and the intercepts in solving a problem; (+) Formulated (all except 1) correct linear equations and functions as models of the given problems; (+) Correctly identified what the given and unknown quantities represent in the linear equations and graphs; (-) Scales used in the graph were inconsistent with the values in the problem; (-) Graphs inappropriately included negative values; (+) Failed to check the graph with the slope in the equation it represents; (-) Had one miscalculation due to carelessness; (+) Correctly wrote a linear equation given two sets of values; (+) Explained clearly the reason why the y-intercept can be found as the constant term in the slope-intercept form of the equation;
Michael	(+) Used directions to describe slopes but incorrectly mentioned downhill as going down towards left; (+) Formulated correct linear equations and functions as models of the given problems; (+) Correctly identified what the given and unknown quantities represent in the linear equations and graphs; (+) Correctly applied slope and intercepts in solving problems; (+) Sketched the graph correctly by considering only realistic values and indicating all necessary labels; (+) Carried out all computations correctly; (+) Knew how to identify but (-) Can't explain why the y-intercept is the constant term of the slope-intercept form of a linear equation.
Sara	(-) Can't differentiate an uphill from the downhill as illustrations for slope; (-) Tried to formulate linear equations and functions as models of the given problems but got 2 wrong equations from the attempts; (-) Graph was incorrect and had no labels; (+) Was able to identify what the given and unknown values represent; (-) Did not understand what the y-intercept is and why it can be found as the constant term in the equation $y = mx + b$.

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Appendix
Paper and Pen Test on Linear Equations and Functions

1. The words "uphill" and "downhill" are relative words in the English language. They depend on where you are standing on a hill AND which way you are looking. How could we use these words to describe some of the lines we graphed and communicate it to everyone in the class?

2. You and a friend are knitting a scarf that will be 72 inches long. Your friend knits the first 24 inches and then gives you the scarf to finish. You expect to knit at a rate of 8 in/day.
   (a) Use the verbal model to write an equation giving the length y of the scarf (in inches) after you have been knitting for x days.

Length of scarf	=	Length knitted by your friend	+	Knitting rate	•	Knitting time

   (b) The knitting rate is none other than the ________________ of the linear equation.
   (c) The length knitted by your friend refers to the _______________ of the line.
   (d) Graph the equation.
   (e) After how many days will you finish the scarf?

3. "Straight line" depreciation is given by D = P – S, where D is the depreciation, P is the purchase price, and S is the salvage value. If a corporation purchases a machine $10,000, graph the equation using D as the vertical axis and S as the horizontal axis.

4. You make and sell birdhouses. Your fixed costs for your tools and workspace are Php3,000. The cost of wood and other materials needed to make a birdhouse is Php10. You sell each birdhouse for Php50. Let x represent the number of birdhouses you make and sell.
   (a) Write a function for your total costs, c(x).
   (b) Write a function for your income, i(x).
   (c) Your profit is the difference of your income and total costs. Write a function for your profit, p(x).
   (d) What is your profit when you make and sell 100 birdhouses?

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(e) You are said to "break even" when your profit is Php0. That is, you neither earn nor lose. How many birdhouses do you need to make and sell in order to break even?

5. A police department finds that the number of crimes (y) committed in one week in a small city depends on the number of police officers (x) on special patrol. If there are 85 crimes committed when no police are on special patrol and the number drops to 75 when two police are on special patrol, write the equation for the relationship.

6. We have been told that any equation in the form $y = mx + b$ is called the slope-intercept form of a straight line where "*m*" is the slope and "*b*" is the y-value where the line crosses the y-axis. Why is "*b*" always the y-intercept?