# Students' Attitudes and Problem Solving with Vee Diagrams

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This paper examines critical issues emerging from a study conducted, over two school terms with groups of Years 7 and 10 students from 3 Australian regional schools, to investigate the impact of using some innovative meta-cognitive strategies on students' problem solving skills and attitudes towards mathematics. Attitudinal data from questionnaires were analyzed using the Rash Model. Qualitative data collected from some Year 7 and Year 10 students enabled a tentative comparison of emerging trends across the two year groups based on their responses to selected questionnaire items, reflective prompts and vee diagrams of mathematical problems. Findings have implications for ways in which innovative ideas in mathematics may be presented to students to ensure a more positive and seamless incorporation into their regular learning in mathematics classrooms at primary and secondary levels.

**Keywords:** problem solving, mathematics attitudes, innovative strategies, meta-cognitive strategies

Learning mathematics more meaningfully and more flexibly, making connections between mathematical ideas and their applications in real life and in problem solving, and developing not only proficiency in solving problems but including as well the development of their critical ability to justify their solutions and strategies in terms of mathematical principles and concepts are some of the fundamental ideas that currently underpin many national curricular reforms in mathematics (AAMT, 2006; NCTM, 2000). Along with these desirable cognitive developments is also the need to develop students' positive attitudes towards mathematics. According to many researchers (Leder, 1987; Zan, Brown, Evans & Hannula, 2006), students who are successful in mathematics often have positive attitudes towards mathematics, and those that consistently fail mathematics have entrenched negative mathematics attitudes. Highly promoted also in innovative, quality and/or productive teaching frameworks (Queensland Department of Education and Training (DET), 2009; New South Wales Department of Education and Training (DET), 2003) is the need to encourage students to work and communicate mathematically through problem solving, making connections, investigation, thinking, reasoning, justifying, proving, and reflecting as part of regular mathematics classroom practices. It is also important that mathematical tasks are interesting and engaging to enable these productive, working and communicating mathematically processes. As Hollingworth, Lokan & McGrae (2003) noted in their video study of Year 8 teachers in a variety of countries, "Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking" (p. xxi). They noted "an over-emphasis on 'correct' use of the 'correct' procedure to obtain 'the' correct answer. Opportunities for students to appreciate connections between mathematical ideas and to understand the mathematics behind the problems they are working on are rare" (p.xxii). For this paper, the main focus question is: *Why is it that, in the classroom, when students are provided with the opportunity to be innovative and creative in their own approaches, these are not often readily accepted or welcomed?* This paper reports relevant data from a literacy-numeracy project conducted to assist a group of Years 10 and 7 educationally disadvantaged (ED) students improve their literacy and numeracy learning outcomes as prescribed by the *K-10 Mathematics Syllabus* (NSWBOS, 2002)

Instead of continuing "a syndrome of shallow teaching, where students are asked to follow procedures without reasons, and (where) more than 'shallow teaching' is needed for students' conceptual understanding and problem solving abilities to improve, the project reported here deliberately focused on ED students building on their cognitive structures and deepening of their understanding of the interconnections between mathematics principles and concepts on one hand, and (multiple) methods of solutions on the other, by requiring them to use two meta-cognitive strategies as part of their mathematics learning and problem solving experiences within a classroom setting. The usefulness of these two meta-cognitive tools: vee diagrams and reflective stories, were then examined and the impact these cognitive constructions and developing understanding on students' attitudes towards mathematics and performance with workshops tasks and tests were also monitored. In contrast to their normal mathematics classroom practices, the students that participated in the project used the innovative strategies as means to support their thinking and reasoning and communication of their mathematical understanding and reflections during weekly workshops.

The selection of workshop tasks (i.e., content and problem type) were intended to provoke cognitive conflict while assisting the students to overcome the typical struggles and difficulties often experienced when faced with solving different and unfamiliar tasks. It was anticipated that addressing these difficulties through the appropriate use of guiding questions on vee diagrams and prompts for reflective stories would be a suitable pedagogical approach to provoke the necessary cognitive conflict (Piaget, 1972). The latter should evoke an active reorganization of a student's existing pattern of meanings towards the grasping of meaning, felt-significance that meaning is grasped, and being motivated and choosing to learn mathematics more meaningfully. To encourage and facilitate these processes, students were encouraged and supported to actively engage with completing vee diagrams and composing reflective stories. During the project, it was further expected that students' attitudes towards mathematics would be influenced by their performance with workshop tasks and tests.

## Theoretical Principles

Whilst the constructivist perspective promotes meaningful learning and the active engagement of students in constructing their own meanings based on their learning experiences (Piaget, 1972; Ausubel, 2000), Gowin's educating theory (1981) proposes that the ultimate goal of teaching is the *achievement of shared meaning*. By using *educative materials of the curriculum*, the teacher and student aim at congruence of meaning; the teacher acts intentionally to change the meaning of the student's experience, using curriculum materials. In a moment of choosing to pay attention to the teacher and the materials, the student acts intentionally to *grasp meaning. The aim is shared meaning*. Interactions and negotiations of meanings between the student and the teacher can be brief or can last a long time, but the aim is to *achieve shared meaning*. In this interaction, both teacher and student have definite responsibilities.

For the teacher, s/he is responsible for seeing to it that the meanings of the materials the student grasps are the meanings the teacher intended for the student to take away (see also Thompson & Salandha, 2003). The student, on the other hand, is responsible for seeing to it that the grasped meanings are the ones the teacher intended. When these *separate responsibilities are fulfilled* and shared meaning is achieved, an *episode of teaching has happened*.

After teaching has resulted in shared meaning, the student is ready to decide whether to learn or not. *Choosing to learn a grasped meaning is a responsibility of the learner and cannot be shared*. Each learner is responsible for his/her own learning.

A vee diagram, as introduced by Gowin, is an epistemological tool which explicates the principles of his educating theory and a means of guiding the *thinking* and *reflections* involved in *making connections* between the *conceptual structure* of a discipline on one hand, and its *methods of inquiry* on the other, as required for the investigation and analysis of a phenomenon or event to generate new knowledge claims as answers to some focus questions. A completed vee diagram would therefore provide a record of the conceptual and methodological analyses of a phenomenon/event to generate new knowledge. To guide the *thinking* and *reasoning* involved in *mathematical problem solving*, the original epistemological vee (see Gowin, 1981) was later modified by Afamasaga-Fuata'i (2005, 1998) as illustrated in Figure 1 (Appendix A).

The vee's left side, the "*Thinking*" side, depicts the philosophy or personal beliefs (i. e., *Why I like mathematics?*) and theoretical framework (i.e., principles: "*What general rules and definitions do I know already?*" and concepts: "*What are the important ideas?*") driving the investigation/analysis of a phenomenon/event (e.g., "*Problem*") to answer some focus questions (i.e., "*What are the questions I need to answer?*"). On the vee's right side, the "*Doing*" side, are the records (i.e., "*What is the information given?*"), methods of transforming the records (i.e., "*How do I … find my answers?*") to generate some answers or new knowledge claims (i.e., "*What are* *my answers to the [focus] questions?*”) and value claims (i.e., through reflection: “*What are the most useful things I have learnt?*”).

Uses of vee diagrams as assessment tools of students’ conceptual understanding have been examined over time in the sciences (Novak & Cañas, 2006; Mintzes, Wandersee & Novak, 2000) and mathematics (Afamasaga-Fuata’i, 2008, 2005, 1998).

**Mathematics Attitudes**

Efforts in the classroom to redress the common societal perception that “mathematics is difficult” are often exacerbated no less due to the already entrenched attitudes and feelings that students have by the time they reach secondary level. Kloosterman & Gorman (1990) suggest that the formation of the belief that some students learn more readily than others and not everyone will be high achievers in school can lead to a notion that affects achievement in mathematics: the notion that it makes little sense to put forth effort when it does not produce results that are considered desirable. Also affecting learning and attitude are other factors such as motivation, the quality of instruction, time-on-task, and classroom conversations (Hammond & Vincent, 1998; Reynolds & Walberg, 1992) and as a result of social interactions with their peers (Reynolds & Walberg, 1992; Taylor, 1992)

Many studies have been conducted on mathematics attitudes and teaching (Leder, 1987; McLeod, 1992; Zan, Brown, Evans, & Hannula, 2006) but for the purposes of this project, McLeod’s (1992) definition of attitudes is adopted: “*affective responses that involve positive or negative feelings of moderate intensity and reasonable stability*” (p. 581). McLeod contends that attitudes develop with time and experience and are reasonably stable, so that hardened changes in students’ attitudes may have a long-lasting effect. Lefton (1997) also argues that attitude is a learned pre-disposition to respond in a consistently favourable or unfavourable manner towards a given object. Positive and negative experiences of school activities produce learned responses which may in turn impact on students’ attitudes as they get older, when positive attitudes towards mathematics appear to weaken (Dossey, Mullis, Lindquist, & Chambers, 1988). Awareness of these complex interacting factors informed the research project in relation to the potential impact of the innovative meta-cognitive strategies on students’ attitudes, motivation to complete mathematical tasks and subsequent mathematics performance.

## Method

Using a single-group research design, the project monitored the impact the usage of vee diagrams and reflective stories had on students’ mathematics attitudes and abilities to solve mathematics tasks. First, to determine changes in student’s attitudes to mathematics, a pretest-posttest research strategy was used by administering questionnaires at the beginning and end of the project. Second, to track students’ mathematics competence during the project, an interrupted time series design model was utilised by administering a diagnostic test three times (beginning, middle and end of project) to establish students’ ability estimates at three different points. Third, weekly workshops were offered, which required students to apply vee diagrams to guide the analyses of tasks and to display both the solutions and conceptual bases of methods used. Fourth, students used their self-created concept lists from vee diagrams (vees) and/or given prompts, on a "*Tell Your Own Story*" (TYOS) sheet, to compose reflective responses of their mathematics experiences. Data from students' vee diagrams and reflective responses provided additional evidence to substantiate test results and document students' increasing proficiency or otherwise, using the innovative tools. The researcher also kept an anecdotal diary of the workshops.

A total of 32 Year 10 students took the pre-questionnaire (pre-Q) from 3 regional Australian schools (coded A, B and C) while 22 Year 10 students took the post-questionnaire (post-Q) from Schools A and B only. Approximately forty four (44) workshops throughout Terms 2 and 3, 2007 were offered, as extra assistance in mathematics. A workshop was held at least once a week over two school terms for each school (A, B and C). The researcher participated in all workshops assisted variously by student teachers (in Schools B and C) and in addition, a school teacher in School A.

Since the project deliberately set out to introduce vee diagrams and reflective compositions, educating the students to use this innovative approach, required that the teacher and students socially interacted and negotiated meaning to achieve a convergence of shared meaning about what needed to be done. The first workshop therefore focused more on familiarizing students with the vee diagram, its different vee sections and specific guiding questions and demonstrating the process of how a vee diagram may be completed using a simple mathematical task.

Incrementally introducing vee diagrams to students, they were provided with partially completed vee diagrams in which the sections: *Problem, What is the given information?, What are the questions I need to answer?* and *What are the main ideas?* were already completed. Their task was to complete the rest of the sections, namely: *What general rules or definitions do I know already?, How do I use the given information and what I know to find my answers?, What are my answers to the focus questions?, What did I learn as a result of solving the problem?* and *Why do I like mathematics?*.

As students became more familiar with the process of completing vee diagrams, increasingly more of the vee sections were left blank until only the problem and focus question sections were provided whilst students completed the rest. By encouraging students to think and reason from the problem statement (often a word problem), they were guided to identify the main ideas, articulate the relevant general rules and identify appropriate methods to generate solutions and answers for the problem's focus questions.

A similar incremental approach was also implemented with the TYOS prompts; students individually completed their sheets question by question after having a whole class discussion of the necessary process. Consequently, students, over subsequent workshops, progressively completed vee diagrams and TYOS prompts with the researchers and teachers facilitating and scaffolding students' efforts as students and teachers interacted and negotiated meaning towards a convergence of shared meaning. Whilst these joint endeavours did not always result in satisfactory completion of all questions/prompts by the end of a lesson, the social dynamics of classroom dialogue and conversations led to enlightenment about, and clarifications of, the purpose and role of the vee diagram and TYOS strategies in learning about mathematics and solving problems more meaningfully. By the sixth workshop, partially completed VEE DIAGRAMs provided only the *Problem*, *Focus Questions* and *Given Information* entries with the provision of only the *Problem* and *Focus Questions* entries by the ninth workshop until the end of the project, with the students left to complete the rest of the vee sections. This strategy ensured students were incrementally eased into learning about a different and new idea within the first few workshops.

Students were also allowed to discuss their ideas in pairs or groups of 3 but the final completion of VEE DIAGRAMs and TYOS sheets were done individually. Students were also invited to bring along problems to be solved from their normal classes and/or pose their own as challenge tasks for the application of the vee diagram and/or for the others to have a go at solving.

Weekly workshops involved the completion of vee diagrams of problems/activities followed by completion of the *Tell Your Own Story* sheet. The extent to which both of these two tasks were fully completed within one workshop differed between workshops as it was dependent on the time of day and prior classes students attended. It was not uncommon for students to take anywhere from 5 to 10 minutes of a 50 minute lesson to finally settle down to work particularly in the afternoon classes.

In general, the focus of each workshop was primarily for students to routinely develop and reinforce the processes of asking the guiding questions; reasoning with the given task description; identifying given information; reflecting upon their current knowledge in order to identify relevant concepts, principles and procedures; reflecting upon their solutions to formulate their value and philosophical claims; and then finally communicating their constructed meanings through entries in their vee diagrams and responses to TYOS prompts. Whilst the processes of thinking, reasoning and reflection were continually reinforced with each subsequent workshop session, students were also encouraged to consider how the same strategies could be applied in their normal classes.

**Data Analysis**

Response categories for the questionnaires items were scored 1 to 5 in increasingly levels of positive attitudes (variable) towards mathematics whereas responses to test items were scored 0 (incorrect) and correct (either 1 or 2) to indicate zero, partial and/or full credit in successfully completing an item (i.e., variable: mathematical competence). These responses were then added across items to give each person a total score to summarise a student's responses to all items. A person with a higher total score than another is deemed to show more of the variable assessed. Summing the item scores to give a single score for a person implied that the items were intended to measure a single variable, often called a *unidimensional variable*.

Analyses of questionnaire and test data were conducted using the Rasch Unidimensional Measurement Model (RUMM) (Rasch, 1980) and the software Quest (Adams & Khoo, 1996). The Rasch model is the only item response theory (IRT) model in which the total score across the items characterizes a person totally. It is also the simplest of such models having the minimum of parameters for the person (just one) and just one parameter corresponding to each category of an item (generally referred to as a threshold). The case where the response categories are the same across items (e.g., SD, SD, N, SA, VSA) the Rasch model has been called the “rating scale model”; the case where the response categories are different across items has been called the “partial credit model” (e.g., 1 or 2 marks). However, the literature (Rasch Analysis, 2005; Bond & Fox, 2001) argue that the structure and response process for a person responding to an item is identical in the two specifications. Rather than emphasizing two models for the above different specifications, it is more efficient to refer to one RUMM with different numbers of categories and different parameterizations. For a dichotomous item, there is just one threshold whilst there is two in the case of three ordered categories (Rasch Analysis, 2005).

The Rasch Model, where the total score totally summarises a person’s standing on a variable, is based on a fundamental requirement: *that the comparison of two people is independent of which items may be used with the set of items assessing the same variable* (Rasch Analysis, 2005; Bond & Fox, 2001). Thus the Rasch model is taken as a criterion for the structure of the responses which they should be satisfied, rather than a mere statistical description. For example, comparison of the performance of two students’ work marked by different graders should be independent of the graders. In this case, it is considered that the researcher is deliberately developing items that are valid for the purpose and that meet the Rasch requirements (Rasch Analysis, 2005). The Rasch Model provides a range of details such as infit mean squares [ims] and standardized infit *t* [infit *t*] for the purpose of testing the fit of the data to the model (i.e., *test of fit*). For example, if item infit and outfit mean square values lie within a specified range (often 0.83 to 1.20) around 1.00, then items are accepted as fitting the model. If not, then a theoretical consideration against the purpose of the test/questionnaire is necessary to determine whether or not to delete the item from further analysis. If the data fit the model, the Rasch analysis linearises the score, bounded by 0 and the maximum score on the items, into measurements. This linearised value in logits determines the *location of the person* on the unidimensional continuum (Rasch Analysis, 2005; Bond & Fox, 2001).

The model is therefore paramount and misfit suggests the items or questions are not working together consistently to define an interpretable construct. Hence, evaluation of the fit of the data to a Rasch model provides information about the coherence of items to measure the underlying theoretical variable or construct. Separation reliability indices are also provided by the Rasch analysis to indicate how well the items and persons worked consistently to produce valid measures of the underlying variable.

For the project, thirty four (34) items in the attitude questionnaire used a 5-point Likert scale with response categories ranging from Very Strongly Agree (VSA), Strongly Agree (SA), Neutral (N), Strongly Disagree (SD) to Very Strongly Disagree (VSD) with marks for test items ranging from zero (incorrect) up to 2 marks to represent partial and/or full credit for different items. Subsequently, all questionnaire and test results were analysed using the RUMM.

The response data from the vee diagrams and reflective stories, on the other hand, were analysed qualitatively. Students' responses were collated and recorded in a spreadsheet in preparation for the identification of emerging main categories and subsequent development of a framework to organize its presentation in a more meaningful manner.

For this paper, the initial analysis of students' attitudes reported in Afamasaga-Fuata'i (2009) was extended further to examine the type and nature of items that shifted (positively and negatively) over time. In addition, a comparison of students' responses to the single open question in the questionnaire: Item 35 (*I intend to take mathematics next year*) was conducted to provide additional data to specifically answer this paper's focus questions. A copy of the questionnaire is in Figure 2 (Appendix A).

## Results

Results from the extended analysis of questionnaire data is presented first followed by those from the open questionnaire item about students' intention to continue studying mathematics in the following year at Year 11.

Rasch analyses of students' responses to questionnaire items showed the overall fit for both items and persons, on average, was acceptable. An inspection of each item's fit however showed 6 items were outside the acceptable limit; consequently, the 6 misfit items were deleted from further analysis. Subsequent analysis confirmed all ims values were within the acceptable limit, thereby corroborating the fit of the data to the Rasch model. Findings (adequate fit of the data to the model, high person separation index and high Cronbach alpha) collectively indicated all items worked together to define and measure a single underlying construct and the persons who attempted the items performed in expected ways (see Afamasaga-Fuata'i, 2009). For example, those with positive attitudes were separated out along the continuum towards the top and those with negative attitudes towards the bottom.

Pre-questionnaire (pre-Q) and post-questionnaire (post-Q) comparisons (Table 1, Appendix A) indicated no statistically significant, difference in attitudes between administrations ($p$=0.18) and a small effect size (0.16). One way of interpreting the construct: "mathematics attitudes" as measured by the questionnaire items, is to analyse the items that were clustered at the top (positive attitudes) and bottom (negative attitudes) ends of the variable maps for the pre- and post-questionnaires (see Figure 3, Appendix A). The results of such an analysis indicated that "positive mathematics attitudes appeared variously described by strong feelings of liking, interest, enjoyment, intellectual challenge, not worrying and not being nervous when doing mathematics, the promotion of creative thinking and development of flexible methods of solution as calibrated for the pre-Q variable map. In contrast, by the end of the project, positive mathematics attitudes continued to be holistically described by these strong feelings and intellectual challenge and in addition, mathematics becoming a most favourite subject, perceiving teacher assistance (or scaffolding) as a positive requirement, being convinced that best strategies for flexibly solving problems and learning mathematics can result from a conceptual understanding of methods used and that these skills have cross-curricular value. Collectively, these positive descriptors (in terms of thinking, feeling, and acting in mathematics) lent support to their strong belief in their own abilities and potential to learn advanced mathematics. In contrast, poor mathematics attitudes at the beginning of the project appeared described by strong negative feelings (e.g., nervousness, blank minds, and strong dislike) and negative actions (e.g., resorting to memorization strategies in problem solving) and strong disagreement about the perceived usefulness of mathematics for a successful life. By the end of the project, some of these concerns became more positive except for the real-life relevance and mathematics' cross-curricular usefulness. In addition, negative attitudes were newly indicated by disagreements with the role of models/diagrams in problem solving, students' willingness to improve their mathematical understanding, continuing disenchantment with mathematics, and perceived irrelevance in understanding newspaper reports and finance graphs" (Afamasaga-Fuata'i, 2009, p.36).

For this paper, further analysis of the pre-Q and post-Q item estimates was conducted to identify the nature of the shifts of item estimates (or locations on the variable maps) between the two administrations, and any emerging categories. Consequently, provided in Table 2 is a comparison of item estimates (Columns 2 & 3), including standardized differences ($\delta_1$ - $\delta_2$) (Column 7) for the 14 items with complete data sets. An inspection of Column 9 revealed that only two items had differences that were statistically significant (e.g., Items 22: *Mathematics is not my strength, I avoid it whenever I can* [$p$=0.01] and Item 28*: To succeed in life you need to be able to do mathematics* [$p$=0.05]). For the former, the post-Q attitude estimate was relatively more negative while it was relatively more positive for the latter item. This might be interpreted to mean that students do realise the usefulness of mathematics in life but their efforts (or lack thereof) are unfortunately hampered by their current inability to do well in mathematics.

Further examination of the standardized differences (std'ised [$\delta_1$ - $\delta_2$]), Column 7, Table 2) indicated two categories, namely positive and negative values. Labelling these categories as those that "became more negative" (positive std'ised [$\delta_1$ - $\delta_2$]) and those that "became more positive" (negative std'ised [$\delta_1$ - $\delta_2$]) provided the results shown in Table 3. It appeared that Column 1 items (Table 3) indicate that project experiences may have influenced some ED students' attitudes to be more entrenched and/or gravitate towards the negative direction (i.e., item estimates became more negative). More particularly, project experiences for some ED students may have further reinforced that (i) mathematics is not their strength; (ii) they do not always enjoy studying mathematics; (iii) mathematics is not needed for understanding newspaper reports and graphs; (iv) they often need teacher support, (v) mathematics is not their favourite subject; (vi) they have blank minds and are unable to think when doing mathematics; (vii) they often avoid problems if they are unable to solve; and (viii) rules learnt in previous classes are forgotten.

Table 2

*Year 10 Cohort Comparison of Item Estimates from the Two Questionnaires*

| ITEM NAME | Delta | | Adjusted Delta | | Difference | | Chi-SQ | p |
|---|---|---|---|---|---|---|---|---|
| | preq | postq | preq d1 | postq d2 | d1-d2 | d1-d2 std'ised | | |
| item 1 | 0.29 | 0.30 | 0.27 | 0.32 | -0.05 | -0.15 | 0.02 | .88 |
| | 0.22 | 0.22 | | | | | | |
| item 2 | -0.11 | -0.16 | -0.13 | -0.14 | 0.01 | 0.05 | 0.00 | .96 |
| | 0.19 | 0.19 | | | | | | |
| item 3 | -0.01 | 0.09 | -0.03 | 0.10 | -0.14 | -0.45 | 0.21 | .65 |
| | 0.21 | 0.22 | | | | | | |
| item 5 | 0.14 | -0.43 | 0.12 | -0.42 | 0.53 | 1.90 | 3.60 | .06 |
| | 0.18 | 0.22 | | | | | | |
| item 6 | -0.10 | -0.06 | -0.13 | -0.05 | -0.07 | -0.27 | 0.08 | .78 |
| | 0.19 | 0.19 | | | | | | |
| item 12 | 0.12 | 0.22 | 0.10 | 0.23 | -0.13 | -0.42 | 0.18 | .67 |
| | 0.18 | 0.26 | | | | | | |
| item 14 | 0.03 | 0.05 | 0.01 | 0.06 | -0.05 | -0.18 | 0.03 | .86 |
| | 0.19 | 0.22 | | | | | | |
| item 15 | -0.38 | 0.13 | -0.41 | 0.14 | -0.55 | -1.77 | 3.13 | .08 |
| | 0.21 | 0.22 | | | | | | |
| item 16 | 0.39 | 0.35 | 0.36 | 0.36 | 0.01 | 0.03 | 0.00 | .98 |
| | 0.17 | 0.20 | | | | | | |
| item 19 | 0.59 | 0.54 | 0.57 | 0.55 | 0.02 | 0.08 | 0.01 | .94 |
| | 0.18 | 0.20 | | | | | | |
| item 22 | 0.32 | -0.36 | 0.30 | -0.34 | 0.64 | 2.45 | 6.01 | .01* |
| | 0.17 | 0.20 | | | | | | |
| item 28 | -0.53 | -0.04 | -0.55 | -0.03 | -0.52 | -1.95 | 3.81 | .05* |
| | 0.19 | 0.19 | | | | | | |
| item 29 | -0.29 | -0.52 | -0.31 | -0.50 | 0.19 | 0.55 | 0.30 | .59 |
| | 0.22 | 0.28 | | | | | | |
| item 33 | -0.15 | -0.28 | -0.17 | -0.27 | 0.10 | 0.35 | 0.12 | .73 |
| | 0.19 | 0.22 | | | | | | |
| Means | 0.02 | -0.01 | 0.00 | 0.00 | ChiSQ=17.49(df=13,p=0.18) | | | |

*signficant

In contrast to Column 1 items, those in Column 2 (Table 3) suggest project experiences may have influenced students' attitudes to be more positive. For example, item estimates' shifts in the positive direction demonstrate that students' increased belief that (i) one needs to be able to do mathematics to succeed in life; (ii) mathematics is learnt by understanding the main ideas not memorise rules and procedures; (iii) they like solving mathematics problems; (iv) they do not feel nervous in mathematics classes and they feel they can do the mathematics; (v) understanding mathematics means more than simply memorizing steps and formulas; and that (vi) mathematics is interesting and mathematics classes are enjoyable.

Overall, the extended analysis revealed that the "became more negative" items tended to be the ones that reinforced negative attitudes whilst "became more positive" items plausibly reflected some influence of the innovative, meta-cognitive and problem solving strategies particularly in expanding students' perception of mathematics learning beyond a "doing and rote memorization" view to one that emphasizes the importance of understanding the main ideas that underpin methods, procedures and formulas.

Table 3
*Standardised Differences (Pre-Q $\delta_1$ estimate – Post-Q $\delta_2$ estimate) $\delta_1$ - $\delta_2$*

| Became more negative | | Became more positive | |
|---|---|---|---|
| Item 22: Mathematics is not my strength and I avoid it whenever I can. | 2.45* | Item 28: To succeed in life you need to be able to do mathematics. | -1.95* |
| Item 5: I have always enjoyed studying mathematics in school. | 1.90 | Item 15: I learn mathematics by understanding the main ideas, not by memorizing the rules and steps in a procedure. | -1.77 |
| Item 29: Mathematics is needed in understanding newspaper reports and finance graphs. | 0.55 | Item 3: I like solving mathematics problems. | -0.45 |
| Item 12: Most of the time, I need help from the teacher before I can solve a problem | 0.10 | Item 6: I am nervous in mathematics classes because I feel I cannot do mathematics | -0.27 |
| Item 19: My most favourite subject is mathematics. | 0.08 | Item 33: I do not have to understand mathematics, I simply memorise the steps to solve a problem. | -0.17 |
| Item 2: When doing mathematics, my mind goes blank, and I am unable to think clearly. | 0.05 | Item 1: Mathematics is very interesting to me and I enjoy my mathematics classes. | -0.15 |
| Item 16:If I cannot solve a mathematics problem, I just ignore it. | 0.03 | | |
| Item 14: I have forgotten many of the mathematical rules I learnt in previous mathematics classes. | 0.01 | | |

*significant (see Table 2)

## Year 10: To continue or not with mathematics in the following year

Pre-questionnaire responses to Item 35 (*I intend to take mathematics next year*) showed that 60.7% (n=33) of the students said they would compared to 50% (n=22) by the post-questionnaire. Students' explanations for their decisions appeared influenced by their perceptions about mathematics' utilitarian value, love, dislike of the subject and/or no reward for effort as briefly described below and illustrated by supporting quotes in Table 4.

Table 4
*Main Categories of Student Explanations for Continuing with Mathematics*

| Main Category | Supporting Quotes |
|---|---|
| **Utilitarian value** | *Because maths will help later on in life with general things.*<br>*Because it is useful.*<br>*Because it's useful to learn & know about mathematics. It will help me in latter life with jobs and paying the bills.*<br>*Because I really need a bit of help because I may need it in life e.g., bills.*<br>*Well, I would like to be a builder. So if I don't get a job at the end of the year as a builder, I will (continue with mathematics).*<br>*In case I want to do a job in the future that involves maths.*<br>*Yes Because I will need it when I get a job in the future.*<br>*I am continuing maths because I am going to need it in every day use when I am older.*<br>*Yes I stay on because it will help maybe later on for careers.*<br>*Only for year 11. I will probably drop it from yr 12 because I don't like it and I'm bad at it.*<br>*It depends on the career path I want to take. I might need or not need it. But only because Mum says I have to. I don't like it very much unless I understand.*<br><br>*I don't need maths for what I'm hoping to be later down the track.*<br>*Maths is not a requirement I need for my job.* |
| **Love of mathematics** | *Yes because I believe I need to improve my understanding of this subject.*<br>*To help me a bit more. Because I want to learn and understand maths more.*<br>*To improve my learning skills and to improve at maths.*<br>*Yes Because I want to get better at maths.*<br>*I will take it next year to have the basic understanding of maths and I feel that it is important.*<br>*I think mathematics is a very useful skill & can be fun.*<br>*Because I would like to learn more about it and understand it better.*<br>*I need to improve.*<br>*Cos I enjoy maths.*<br>*I intend to continue maths because it's fun sometimes.*<br>*Because I like it.*<br>*I really enjoy maths its one of my favourite subjects.*<br>*I can learn by the teacher explaining what I have to do & to be with me along the way. I learn heaps also by talking to people about it.*<br>*Because I really enjoy it all the time & my friends do too. Also that because it's the bestest subject that I do. But sometimes I feel that I'm not doing very good.* |
| **Dislike of mathematics** | *I hate maths and I can't do it.*<br>*Because I don't really want to.*<br>*Because I don't want to.*<br>*No because I don't enjoy it.*<br>*I don't really like maths, even if I was good at it.*<br>*No I hate maths so I am not doing it next year.*<br>*Probably won't take it because its on the same line as another subject I wanna do.* |
| **No reward for effort** | *Because I'm not a very good mathematician and I don't enjoy it that much.*<br>*I really want to but I'm in the bottom maths & don't get very good marks & I feel that might do better in another subject, because I'm bad at Mathematics.*<br>*Because I am failing and I don't wish to do maths in school next year because I think it will be a waste of time and I won't pass.*<br>*I find maths more difficult than any other subject and would not do it over something I like doing.* |

**Utilitarian value** – Students reasoned that they would continue to take mathematics because it is useful to learn and know for general things they do in life (e.g., paying bills, a future job as a builder, or careers in general) or because of their parents' expectations.

**Love of mathematics -** Students justified their continuation with mathematics intrinsically, based on their interest, enjoyment and liking for the subject thus they argued that it is important to continue learning, understanding and knowing more of it.

**Dislike of mathematics** – Students' decision not to continue with mathematics is emotively or experientially rationalized in terms of their negative feelings towards the subject, perceived inability to succeed in spite of efforts to perform better, apparent irrelevance to future career aspirations, or timetable clashes with another preferred, more likeable subject.

**No reward for effort** – Students recognize their own mathematical abilities based on previous and current experiences and concluded that there was no sense in taking a subject which they know they will not pass.

In addition to the listed categories, some students were ambivalent about their intentions to continue as reflected by responses such as "*Maybe I don't know yet*" and *"Don't know haven't thought about it."* Other students opted to discontinue studying mathematics because they were leaving school at Year 10 (e.g., "*I'm not coming back next year so I'm not doing it.*"). Overall, it appeared that this group was split between continuing on with mathematics and discontinuing it after Year 10.

## Discussion and Main Findings

Students' responses and subsequent analyses as presented in this paper was an attempt to answer the paper's focus question, namely, *Why is it that, in the classroom, when students are provided with the opportunity to be innovative and creative in their own approaches, these are not often readily accepted or welcomed?* The discussion below is organized around the main findings resulting from the extended analysis of questionnaire items and examination of student responses to the open question on continuation of further studies in mathematics. Overall, the literacy-numeracy project specifically examined the impact of two innovative meta-cognitive strategies on students' achievement of numeracy and literacy outcomes and attitudes towards mathematics; the latter being the focus of this paper. For this paper, the author was particularly interested in examining why students, instead of readily embracing the new innovation, initially struggled and tended not to sustain interest in it. Hence, to provide some answers, (1) the variations in item estimates between the pre- and post-Q administrations and (2) students' reasons for continuing/discontinuing further studies in mathematics were examined.

Item estimate variations – The shift in item estimates and the resulting two categories demonstrate that the innovative, meta-cognitive strategies introduced and implemented in the workshops influenced students' mathematics perceptions and attitudes two different ways.

First, a positive influence was demonstrated by items directly reflecting the fundamental ideas and processes promoted through the completion of vee diagrams and reflective stories, namely, identifying main ideas underpinning methods, procedures and formulas; developing a much broader (and more conceptual) perception of learning mathematics beyond simply doing the problem and rote memorizing formulas and procedures; enjoyment in solving problems; and perceiving a strong association between their mathematical ability and success in life.

Second, a negative influence was demonstrated with items describing avoidance actions; inability to think clearly; and acceptance of their abilities to do mathematics when problems are difficult. Item variations also confirmed their dependence on teacher support for guidance; lack of enjoyment studying school mathematics; their least-liked subject; and not needed for understanding newspaper reports and graphs.

Overall, the results demonstrated that, as a consequence of the innovative project, some students continued to develop more positive attitudes whilst others became more entrenched in their negative attitudes towards mathematics. Given the short-term duration of the innovative project, and as pointed out by McCleod (1997), attitudes develop with time and experience and are reasonably stable so that hardened changes may have long lasting effect. It is therefore reasonable, one hand, to expect that for students, who have had consistently poor experiences in mathematics, that they would take a much longer time (than the duration of the project) for their entrenched, negative attitudes towards mathematics to change. On the other, students who already feel positive about, and confident in, their mathematical abilities appeared to have even more positive attitudes as a result of the promotion of student autonomy in their own learning and development of their own mathematical ideas and ways of knowing and learning about mathematics as they completed vee diagrams and composed reflective responses in the project. In relation to the paper's focus question, the findings demonstrated that the nature of students' responses to the attitudinal questionnaire appeared to have been much influenced by their regular classroom practices with subsequent beliefs and attitudes (positive and negative) enculturated as a result of these experiences and that their participation in the short-term project appeared to push them further in the direction of their developed attitudes and perceptions.

Continuation with the Study of Mathematics - In the light of the above discussion of students' entrenched and developed perceptions about, and attitudes towards, mathematics learning, the emerging categories of students' reasons for continuing or discontinuing mathematics studies demonstrated that for about half of the students, despite their expressed attitudes (especially if they were negative) they still preferred to continue studying mathematics in the following year for utilitarian reasons and/or their love of mathematics. Some students recognize that they need to improve their performance and learn more mathematics. The other half of the students explicitly preferred not to continue with mathematics because they do not like the subject and they saw no reason to continue studying a subject that leads to failure and provides little reward for a lot of effort. Some students also indicated they would leave school at the end of Year 10.

## Implications

The main findings presented above from the two sets of analyses have implications for teaching and learning mathematics and the evidence-based need to put in place innovative programs to inspire students much earlier than Year 10. Furthermore, the usefulness of vee diagrams as an epistemological tool for thinking, reasoning, justification and reflection should be made more explicit to students much earlier than Year 10 to enculturate some positive perceptions of their own learning and love of mathematics. Allowing students to communicate the benefits of vee diagrams in problem solving could draw students' attention to the potential of vee diagrams to systematically guide their thinking, reasoning, justification, reflection and communication during and after problem solving. It was evident that students appreciate and enjoy solving mathematics problems that provide them with a positive learning experience and feeling of significance that they have understood the new meaning; therefore teachers should explicitly encourage students to think, reason, make connections to their existing knowledge, reflect on their learning and communicate their new meanings during and after problem solving experiences. Teacher-led discussions which draw out the educational value of thinking, reasoning, justification, and reflection would be useful in promoting a more comprehensive and conceptual view of doing mathematics and hence more positive attitudes towards mathematics. Teachers should be encouraged to support the use of vee diagrams and reflective prompts in the classroom to support students' thinking, reasoning, justification, reflection and communication of their mathematical learning as promoted through productive and quality teaching and learning frameworks.

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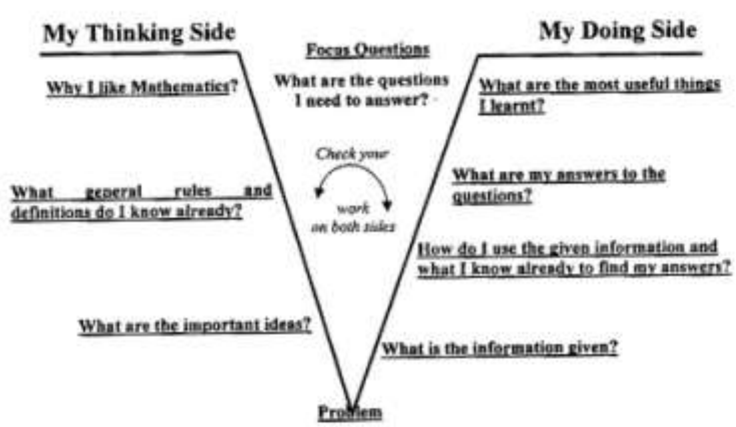
**Appendix 1**
**Mathematics Problem Solving Vee Diagram**



## Appendix 2
## Mathematics Attitude Questionnaire

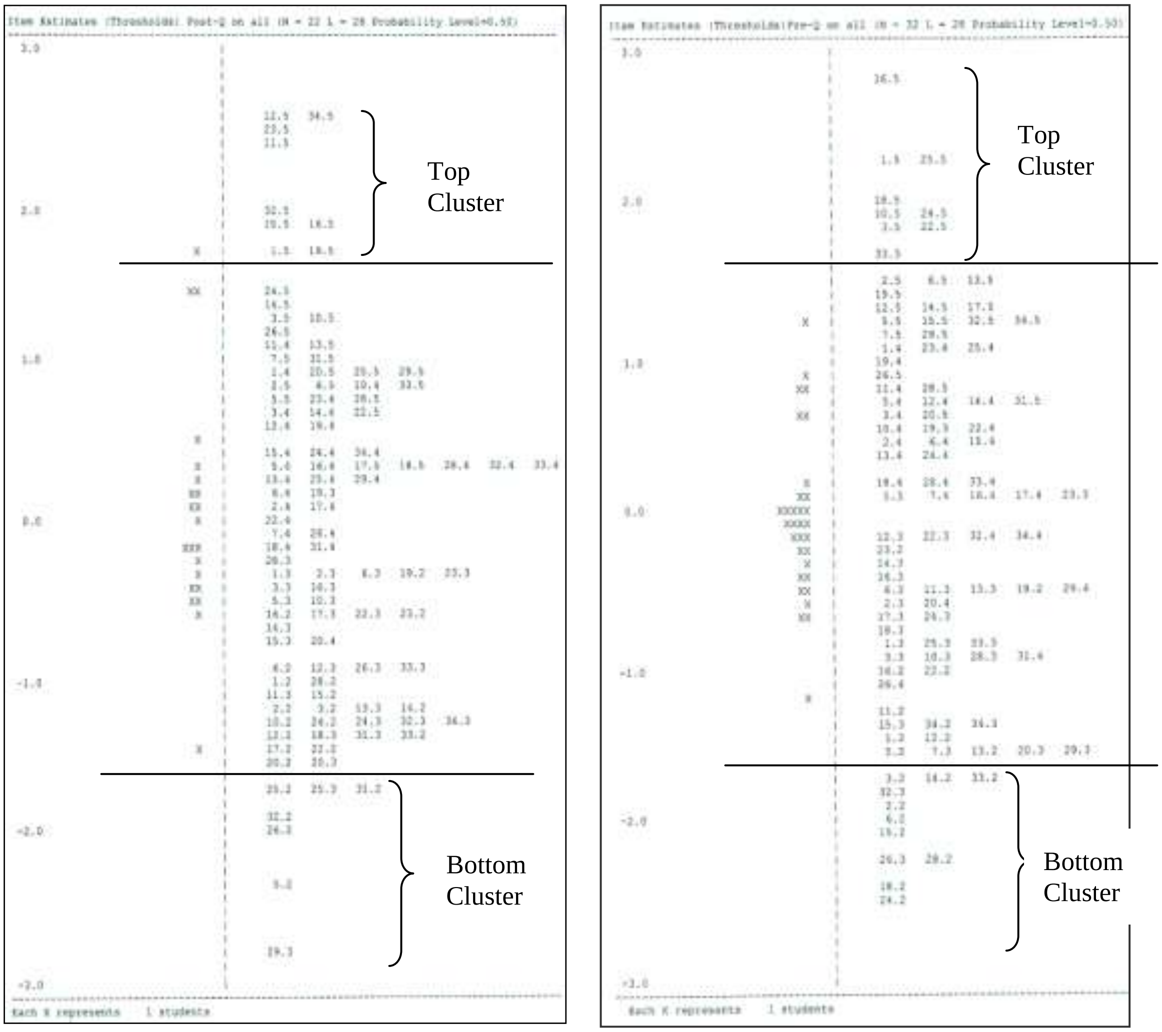
1. Mathematics is very interesting to me and I enjoy my mathematics classes.
2. When doing mathematics, my mind goes blank, and I am not able to think clearly.
3. I like solving mathematics problems.
4. Mathematics makes me feel uncomfortable and impatient.
5. I have always enjoyed studying mathematics in school.
6. I am nervous in mathematics classes because I feel I cannot do mathematics.
7. It makes me nervous to even think about having solving a mathematics problem.
8. I really like mathematics; it's enjoyable.
9. I can cope with a new problem because I am good in mathematics.
10. I get worried when solving a problem that is different from the ones done in class.
11. I can find many different ways of solving a particular mathematics problem.
12. Most of the time, I need help from the teacher before I can solve a problem.
13. I believe that if I use what I know already, I can solve any mathematics problem.
14. I have forgotten many of the mathematical concepts that I have learnt in previous mathematics classes.
15. I learn mathematics by understanding the main ideas, not by memorizing the rules and steps in a procedure.
16. If I cannot solve a mathematics problem, I just ignore it.
17. Successfully solving a problem on my own provides satisfaction similar to winning a game.
18. I feel nervous when doing mathematics.
19. My most favourite subject is mathematics.
20. Mathematics classes provide the opportunity to learn skills that are useful in daily living.
21. To succeed in school, you don't need to be good in mathematics.
22. Mathematics is not my strength and I avoid it whenever I can.
23. I don't think I could learn advanced mathematics, even if I really tried.
24. Doing mathematics encourages me to think creatively.
25. I learn to think more clearly in mathematics if I make a model or draw diagrams of the problem.
26. Mathematics is important for most jobs and careers.
27. Solving mathematics problems helps me learn to think and reason better.
28. To succeed in life you need to be able to do mathematics.
29. Mathematics is needed in understanding newspaper reports and finance graphs.
30. Communicating with other students helps me have a better attitude towards mathematics.
31. I am interested and willing to improve my understanding of mathematics.
32. The skills I learn in mathematics will help me in other subjects at school.
33. I do not have to understand mathematics, I simply memorise the steps to solve a problem.
34. I learn mathematics well if I understand the reasons behind the methods used.
35. I intend to continue taking mathematics next year.

## Appendix 3
## Rasch Results of the Year 10 Questionnaire Data

| | Pre-Questionnaire | Post-Questionnaire |
|---|---|---|
| **Items** | n = 28 | n = 28* |
| Mean Estimate | 0.00 | 0.00 |
| Standard deviation | 0.36 | 0.37 |
| Item Separation Reliability | 0.00 | 0.00 |
| Items with zero scores | 0 | 0 |
| Items with perfect scores | 0 | 0 |
| Cronbach's Alpha | 0.82 | 0.90 |
| | | |
| **Persons** | n = 32 | n = 22 |
| Mean Attitude Estimates | - 0.03 | 0.05 |
| Standard deviation | 0.49 | 0.69 |
| Effect size | | 0.16 (small) |
| Case Separation Reliability | 0.83 | 0.92 |
| Cases with zero scores | 0 | 0 |
| Cases with perfect scores | 0 | 0 |
| Mean Questionnaire score (s.d.) | 49.63 (11.80) | 55.64 (16.92) |
| Effect size | | 0.52 (medium) |

*the other 6 items did not have complete data sets

## Appendix 4
## Item clusters for the pre- and post- questionnaire data




(a) (b)

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